

Direct Test of the Scalar-Vector Lorentz Structure of the Nucleon- and Antinucleon-Nucleus Potential

R. Hofmann¹, Amand Faessler² and Th. Gutsche²

*Institut für Theoretische Physik, Universität Tübingen
Auf der Morgenstelle 14, D-72076 Tübingen, Germany*

Abstract

Quantum Hadrodynamics in mean field approximation describes the effective nucleon-nucleus potential (about -50 MeV deep) as resulting from a strong repulsive vector (about 400 MeV) and a strong attractive scalar (about -450 MeV) contribution. This scalar-vector Lorentz structure implies a significant lowering of the threshold for $p\bar{p}$ photoproduction on a nucleus by about 850 MeV as compared to the free case since charge conjugation reverses the sign of the vector potential contribution in the equation of motion for the \bar{p} states. It also implies a certain size of the photon induced $p\bar{p}$ pair creation cross section near threshold which is calculated for a target nucleus ^{208}Pb . We also indicate a measurable second signature of the $p\bar{p}$ photoproduction process by estimating the increased cross section for emission of charged pions as a consequence of \bar{p} annihilation within the nucleus.

Keywords: Quantum Hadrodynamics, covariant mean field potentials, photoinduced $p\bar{p}$ pair creation, $p\bar{p}$ pair production threshold

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1 Introduction

In this article we propose a direct method to investigate if Quantum Hadrodynamics (QHD I,II) [1], which is well suited for the description of nuclear properties such as density distributions, binding energies etc., can be extrapolated to the explicit consideration of antinucleons inside heavy nuclei. In its simplest version, that is QHD I, the relativistic nuclear many-body problem is based on a Lagrangian containing nucleon and isoscalar meson (scalar meson σ and vector meson ω) degrees of freedom. In the mean field Hartree approximation [1] nucleon single particle states are determined from a Dirac equation with Lorentz scalar and vector mean field potentials resulting in positive and negative frequency solutions. According to hole theory the physical vacuum is characterized by a completely filled negative energy Dirac sea and the absence of states carrying positive energy. The charge conjugation operation allows for the interpretation of holes in the sea as antiparticles.

While the scalar mean field is attractive for both nucleon and antinucleon states, the potential induced by the vector meson reverses sign under charge conjugation. Hence scalar and vector potentials for the nucleon almost cancel, resulting in an effective potential depth of about -50 MeV, while both contributions add up for the antinucleon generating an effective potential depth of -700 to -900 MeV.

In this article we investigate the consequences of the Lorentz structure of the nucleon/antinucleon ($N\bar{N}$) mean field potentials, as set up in QHD I, on predictions for near

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threshold cross sections of γ induced proton-antiproton ($p\bar{p}$) pair creation. The respective threshold is determined by the energy of the lowest lying \bar{p} bound state and the Fermi energy of the nuclear ground state. As compared to the free case the $p\bar{p}$ pair creation threshold is therefore strongly reduced due to the large attractive potential felt by the \bar{p} (for a schematic description see Fig. 1). At present it is not clear whether the mean field approximation of QHD I delivers a sufficient description of the \bar{N} -nucleus potential since $N\bar{N}$ annihilation processes can generate a dispersive complex contribution to the mean field potential.

By studying the photoproduction of $p\bar{p}$ pairs on a nucleus we directly test the highly contested assumption of a mean field for the description of explicit antinucleons inside the nucleus. The question if this assumption is reasonable or if one is forced to include effects beyond the mean field approach can in principle be answered by experimental tests of the $p\bar{p}$ pair creation process on nuclei.

For the γ -induced $p\bar{p}$ pair creation we consider a process where a low energy proton is emitted into the continuum while in a first step the \bar{p} remains in a bound state in the nucleus. There are, of course, a variety of competing reactions that also lead to final state protons. At a γ -energy of several hundred MeV (hence well below the $p\bar{p}$ pair creation threshold) Intra Nuclear Cascade (INC) calculations for γ absorption on nuclei [7, 8] are able to reproduce the experimental cross sections. For photon energies of about 1000 MeV, lying around the $p\bar{p}$ pair creation threshold, there exists a semiclassical BUU transport model calculation [10] which gives an order of magnitude estimate for the background due to the direct photoemission of protons.

The \bar{p} , produced in a bound state, can subsequently annihilate in the nucleus, thereby producing a definite average number of emitted pions [9]. For the emission of charged pions this number has been calculated in the framework of an INC calculation by Botvina et al. [4] and turned out to be 2.5 for heavy nuclei. Given the calculated γ -induced $p\bar{p}$ pair creation cross section on nuclei both the cross section of proton and pion emission due to the annihilation of the \bar{p} can be estimated. These results are compared to the respective theoretical estimates for the background of protons and pions coming from direct γ -induced emission.

The paper is organized as follows: In the next section we will shortly describe the derivation of the static mean field potentials from the Lagrangian of QHD I [1] and discuss the relevant parts of the nucleon and antinucleon spectrum. Here we consider only the simplest form of QHD that allows for interactions of the nucleon with a neutral scalar and vector meson field while neglecting the electromagnetic and additional vector meson mediated interactions as described in Ref.[2]. In Section 3 we indicate the construction of the S-matrix in first order perturbation theory for the γ -induced creation of a $p\bar{p}$ pair on nuclei, with details of the actual calculations contained in the Appendices. The dependence of the γ -induced $p\bar{p}$ pair creation cross section on the depth of the scalar-vector potential for a given set of quantum numbers and the results for differential and total inclusive cross sections are presented in Section 4. Finally, in Section 5, we summarize the results and give the conclusions.

2 Mean field potentials

The Lagrangian of QHD I [1] (also referred to as Walecka Model) for interacting nucleon (ψ), massive scalar meson (ϕ) and vector meson (V_μ) fields is given by

$$\mathcal{L}_{WM} = \bar{\psi}[\gamma^\mu(i\partial_\mu - g_v V_\mu) - (M - g_s\phi)]\psi + \frac{1}{2}(\partial^\mu\phi\partial_\mu\phi - m_s^2\phi^2)$$

$$-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}m_v^2 V^\mu V_\mu \quad , \quad (1)$$

where the field strength tensor for the vector meson is defined by

$$F^{\mu\nu} = \partial^\mu V^\nu - \partial^\nu V^\mu \quad . \quad (2)$$

According to Ref.[1], in a nuclear system with a large number of nucleons at nuclear saturation density one can consider the meson fields as classical fields V_μ^c, ϕ^c and only quantize the nucleon field. For spherically symmetric, static nuclei the equations of motion for the meson fields are

$$\begin{aligned} (\nabla^2 - m_s^2)\phi^c(r) &= -g_s\rho_S(r) \\ (\nabla^2 - m_v^2)V_0^c(r) &= -g_v j_0(r) \end{aligned} \quad (3)$$

while the spatial part of V_μ^c must vanish due to rotational symmetry. The scalar and current densities of Eq.(3) are the normal ordered expectation values of the corresponding field operators taken in the nuclear Hartree ground state $|F\rangle$

$$\rho_S(r) := \langle F | : \bar{\psi}\psi : | F \rangle, \quad j_\mu(r) := \langle F | : \bar{\psi}\gamma_\mu\psi : | F \rangle \quad . \quad (4)$$

The Dirac equation for the baryon field modes reads

$$[i\gamma^\mu\partial_\mu - g_v\gamma^0 V_0^c(r) - (M - g_s\phi^c(r))]\psi = 0 \quad . \quad (5)$$

Solutions to the coupled system of Eqs.(3) and (5) can be found by the usual Hartree procedure, where experimental information about nuclear bulk properties is used to determine the coupling constants g_v and g_s [1].

In the following we will assume the potential shapes for $\phi^c(r)$ and $V_0^c(r)$ to be approximated by spherical square wells as

$$\begin{aligned} \phi^c(r) &= \begin{cases} S & , & 0 \leq r \leq r_0 \\ 0 & , & r_0 < r \end{cases} \\ V_0^c(r) &= \begin{cases} V & , & 0 \leq r \leq r_0 \\ 0 & , & r_0 < r \end{cases} \end{aligned} \quad (6)$$

Although this is a rough approximation to the more realistic potentials obtained in Ref.[1] it bares the advantage that analytical solutions are obtained for Eq.(5). For a first estimate of the γ -induced $p\bar{p}$ pair creation cross section the approximation of Eq.(6) is sufficient.

The eigenvalues of Eq.(5) are determined by demanding continuity for the single particle solutions ψ at $r = r_0$. For the nucleus ^{208}Pb we choose potential strengths of $S = 400$ MeV, $V = 450$ MeV and a nuclear radius given by

$$r_0 \approx 1.2 \times A^{\frac{1}{3}} \text{ fm} = 7.2 \text{ fm} \quad .$$

The corresponding spectrum of the negative energy solutions of Eq.(5) is indicated in Fig. 2, where the κ denotes the Dirac quantum number. The potential parameter values are compatible with those determined by a Thomas-Fermi approximation for finite systems and a relativistic Hartree calculation as outlined in Refs.[1, 2].

3 S-matrix element for γ -induced $p\bar{p}$ pair creation

In the following we indicate the calculation of the S-matrix element for γ -induced $p\bar{p}$ pair creation to first order in the electromagnetic coupling constant e . Here we assume the proton to be in a continuum state and the antiproton to be produced in a bound state (see Fig. 1). For simplicity we assume that the proton state is approximated by a positive energy plane wave

$$^{(+)}\psi_p^\mu(x) = \sqrt{\frac{M}{E(\vec{p})V}} u(\vec{p}, \mu) e^{-ipx}, \quad E(\vec{p}) := p_0 = \sqrt{\vec{p}^2 + M^2}. \quad (7)$$

The \bar{p} state is associated with a negative energy solution of Eq.(5) and given by

$$^{(-)}\psi_{p'}^\rho(x) = \exp(-iE'x^0) \begin{pmatrix} g(r, E') \omega_\kappa^\rho(\theta, \phi) \\ i f(r, E') \omega_{-\kappa}^\rho(\theta, \phi) \end{pmatrix}, \quad E' < 0. \quad (8)$$

Here ρ denotes the total angular momentum projection, μ the spin projection, p' summarizes energy and Dirac quantum number κ , while p denotes the respective four momentum. With Bjorken-Drell convention for the $u(\vec{p}, \mu)$ spinor, the wave function in Eq.(7) is normalized to unit probability within a volume V . Accordingly, the same normalization is applied to the wave function of Eq.(8), with details given in Appendix A. The S-matrix element for γ -induced $p\bar{p}$ pair creation in first order perturbation theory is then deduced as

$$\begin{aligned} S_{p,p'}^{1,2;\mu;\rho} &= ie \int d^4x \, ^{(+)}\bar{\psi}_p^\mu(x) A_\nu^{1,2}(x) \gamma^\nu \, ^{(-)}\psi_{p'}^\rho(x) \\ &= ie \int d^4x \, A_\nu^{1,2}(x) j_{tr}^\nu. \end{aligned} \quad (9)$$

The potential $A_\nu^{1,2}(x)$ for a free photon propagating in 3-direction can be written in Lorentz gauge in the following form

$$A_\nu^{1,2}(x) = \frac{\varepsilon_\nu^{1,2}}{\sqrt{2} kV} e^{-ik'x}, \quad k'^0 = |\vec{k}'| =: k, \quad \varepsilon_\nu^1 = (0, 1, 0, 0), \quad \varepsilon_\nu^2 = (0, 0, 1, 0), \quad (10)$$

where the four vectors $\varepsilon_\nu^{1,2}$ characterize the two independent transversal polarizations. The potential $A_\nu^{1,2}$ is normalized to energy k for the volume V .

In contrast to the γ -induced $p\bar{p}$ creation in free space the S-matrix element of Eq.(9) does not vanish since the negative energy eigenstate in Eq.(8) has a nontrivial expansion with respect to momentum eigenstates.

Since the created p and \bar{p} carry inner structure the transition current j_{tr}^ν of Eq.(9) has to be modified. By including additional vector meson interactions in the Lagrangian of Eq.(1) the electromagnetic form factor of the nucleon can be simulated via vector meson dominance [3, 2]. We disregard this possibility by using the simple σ - ω -model of QHD I. A minimal correction to the transition current j_{tr}^ν of Eq.(9) that should not be neglected is the contribution of the anomalous magnetic moment $\mu_{a.m.}$ of the proton. For an asymptotically free theory the demand for Lorentz covariance (parity transformations included) and current conservation leads to the following form of the corrected transition current [6]

$$j_{tr, cor}^\nu = \bar{\psi}_q \gamma^\nu \psi_{q'} + \frac{\mu_{a.m.}}{2M} \partial_\lambda \left(\bar{\psi}_q \sigma^{\nu\lambda} \psi_{q'} \right), \quad (11)$$

where q and q' denote the four momenta of the corresponding states. The constant $\mu_{a.m.}$ is determined in the nonrelativistic limit as

$$\mu_{a.m.} = 1.79284. \quad (12)$$

In Appendix B it is shown that Eq.(11) even holds for eigenstates of the Dirac equation with scalar-vector potentials if one demands some plausible properties for $j_{tr, cor}^\nu$. By partial integration we identify

$$\frac{\mu_{a.m.}}{2M} \int d^4x A_\nu \partial_\lambda (\bar{\psi}_p \sigma^{\nu\lambda} \psi_{p'}) = -\frac{\mu_{a.m.}}{2M} \int d^4x (\partial_\lambda A_\nu) \bar{\psi}_p \sigma^{\nu\lambda} \psi_{p'} , \quad (13)$$

which leads to the corrected S-matrix element including the contribution of the anomalous magnetic moment as

$$\begin{aligned} {}^{cor}S_{p,p'}^{1,2;\mu;\rho} &= ie \int d^4x {}^{(+)}\bar{\psi}_p^\mu(x) A_\nu^{1,2}(x) \left[\gamma^\nu + i \frac{\mu_{a.m.}}{2M} k_\lambda \sigma^{\nu\lambda} \right] {}^{(-)}\psi_{p'}^\rho(x) \\ &= ie \int d^4x A_\nu^{1,2}(x) j_{tr, cor}^\nu . \end{aligned} \quad (14)$$

The subsequent standard derivation of the differential cross section $\frac{d\sigma}{d\Omega}$ for γ -induced $p\bar{p}$ pair creation can be found in Appendix C.

4 Results

The sensitivity of the γ -induced $p\bar{p}$ pair creation cross section σ on the depths of the scalar and vector potentials is displayed in Fig. 3. Here the cross section σ is given for a final state proton at fixed energy ($M+8$ MeV, M is the nucleon mass), while the \bar{p} corresponds to the highest lying negative energy state with $\kappa = +1$. In the variation of the potential depths the difference $\Delta := S - V = 50$ MeV is kept fixed and the value of r_0 has been chosen as $r_0 = 7.2$ fm. The two maxima of σ in Fig. 3 correspond to $S = 380$ MeV and $S = 460$ MeV and reflect the maximal overlap of the bound negative energy solution (associated with the lowest bound \bar{p} state) with the continuum wave function for the p state.

The following results for photoproduction of $p\bar{p}$ have been obtained using the mean field potential parameter values introduced in section 2, which is an adaption of the ^{208}Pb nuclear system (compare to Ref.[2]). In Fig. 4 we indicate the differential inclusive cross sections $\frac{d\sigma_{inc}}{d\Omega}$ for $p\bar{p}$ pair creation at γ -energies k of 50 MeV and 30 MeV above threshold energy k_{th} . One obtains $\frac{d\sigma_{inc}}{d\Omega}$ by summing up the contributions of all energetically possible states of the produced $p\bar{p}$ pair. The global maximum of $\frac{d\sigma_{inc}}{d\Omega}$ for $k = k_{th} + 30$ MeV is at $\theta = 0$, while for $k = k_{th} + 50$ MeV we obtain a pronounced global maximum at $\theta \approx \frac{\pi}{4}$.

The integrated inclusive cross section σ_{inc} for direct proton emission due to $p\bar{p}$ pair creation as a function of photon energy k in the range from k_{th} to $k_{th} + 70$ MeV is shown in Fig. 5. For this energy range \bar{p} bound states with Dirac quantum numbers up to $\kappa = +7$ are included. Final state protons with maximal kinetic energy give the largest contribution to σ_{inc} due to favourable phase space (see for example Eq.(32)). At a γ energy k of about 70 MeV above threshold we obtain an inclusive cross section of 10^{-3} mb to 10^{-2} mb for proton emission due to $p\bar{p}$ pair creation. The background σ_{bg}^p of conventional γ -induced proton emission is estimated at 50 mb from a BUU transport model calculation at a photon energy of $k = 1$ GeV [10].

Assuming that in each channel of the annihilation process

$$\bar{p} + N \longrightarrow X$$

there is at least a charged pion in the final state we have for the total inclusive pion emission cross section $\sigma_{inc}^{ch.pion}$ at $k = k_{th} + 70$ MeV

$$\sigma_{tot,inc}^{ch.pion}(k_{th} + 70 \text{ MeV}) \approx \sigma_{inc}(k_{th} + 70 \text{ MeV}) = 10^{-3} \text{ to } 10^{-2} \text{ mb} ,$$

where $\sigma_{tot,inc}^{ch.pion}$ is obtained by summing the inclusive cross sections $\sigma_{inc}^{ch.pion}$ for specific channels over all channels. Beside the increasing pion production total inclusive cross section due to $N\bar{p}$ annihilation in the ^{208}Pb nucleus an additional observable is the average multiplicity of 2.5 emitted charged pions [4], which can in principle be experimentally tested by coincidence measurements. The corresponding background $\sigma_{bg}^{ch.pion}$ of conventional charged pion emission is estimated at 15 mb based on a BUU transport model calculation at a γ -energy of $k = 1$ GeV [10].

5 Conclusions

Quantum Hadrodynamics I [1, 2] solved in mean field Hartree approximation yields a scalar-vector structure for the real N -nucleus potential, where the effective potential depth (≈ -50 MeV) results from the difference of a large repulsive vector (≈ 400 MeV) and a large attractive scalar (≈ 450 MeV) potential. If one takes the simple mean field picture, according to hole theory, the \bar{N} -nucleus potential would effectively be about 850 MeV deep. This approach neglects typical quantum effects of the full theory and by construction any correlations between the nucleons in the nuclear ground state. Already on the level of a two particle problem there are essential contributions to the imaginary and dispersive real part of the antinucleon-nucleon potential due to $N\bar{N}$ -annihilation. Therefore the mean field picture is highly contested.

For example, Teis et al. [12] indicate the lack of unitarity between the real and imaginary part of the \bar{N} self energy and the absence of Fock terms when applying the simple charge conjugation picture to construct the \bar{N} -nucleus potential in the simple Hartree meson mean field approximation. On the contrary, Mishustin et al. [13] and Schaffner et al. [11] argue for spontaneous and induced creation of $N\bar{N}$ pairs subject to scalar-vector nuclear mean fields in heavy ion collisions. We therefore propose a direct test of the Hartree mean field picture, that is of the depths of the scalar and vector \bar{N} -nucleus potentials. This can be done by measuring the near threshold photoproduction of $p\bar{p}$ pairs on a nucleus, here specified for ^{208}Pb .

In this work we calculated the inclusive cross section for γ -induced $p\bar{p}$ pair creation at γ -energies up to about 70 MeV above threshold. Thereby we assume that the Hartree mean field potential is approximated by a square well (adapted to ^{208}Pb), while for the proton continuum states we use undistorted plane waves. The inclusive cross section is of the order of 10^{-3} to 10^{-2} mb for γ -energies of 70 MeV above threshold.

Transport model (BUU) calculations at a photon energy of about 1 GeV [10] for proton emission excluding $p\bar{p}$ pair creation processes estimate the background cross section σ_{bg}^p at about 50 mb for the ^{208}Pb nuclear system. Given σ_{bg}^p , a 4σ error confidence interval and the calculated inclusive cross section of $\sigma_{inc}(70\text{MeV}+k_{th}) \approx 10^{-2}$ mb would at least require 4×10^8 events μ_p of the inclusive reaction

$$\gamma + ^{208}\text{Pb} \longrightarrow p + X$$

to isolate the $p\bar{p}$ creation signal from statistical fluctuations

$$\begin{aligned} \frac{4\sqrt{\mu_p}}{\mu_p} &\leq \frac{\sigma_{inc}(70\text{ MeV} + k_{th})}{\sigma_{bg}^p} \implies \\ \mu_p &\geq \left(\frac{4\sigma_{bg}^p}{\sigma_{inc}(70\text{ MeV} + k_{th})} \right)^2 \end{aligned}$$

$$= 4 \times 10^8 . \quad (15)$$

This amounts to a measuring time of about 30 days if one takes a photon flux per bin (10 MeV) of 10^5 s^{-1} and a ^{208}Pb target with area mass density of 11g/cm^2 . The photon flux is compatible with values obtainable at CEBAF [14].

A second signature for the γ -induced $p\bar{p}$ pair production is the emission of charged pions as produced by the strong annihilation of the \bar{p} with a nucleon of the nucleus. For ^{208}Pb the background cross section $\sigma_{bg}^{ch.pion}$ of the reaction

$$\gamma + ^{208}\text{Pb} \longrightarrow \pi^\pm + X$$

is estimated with the help of a transport model (BUU) calculation [10] to be 15 mb at a γ energy of 1 GeV. This would require an event number of measured charged pions greater than 3.6×10^7 to suppress statistical fluctuations sufficiently. With the above photon flux per bin and target area density the measuring time is estimated at about 9 days. The predictions of the transport model calculation for the average multiplicity of charged pions due to conventionell photon absorption is about 0.35 [10]. On the other hand, an INC calculations for the multiplicity of charged pions due \bar{p} annihilation in the ^{208}Pb nucleus suggests a value of 2.5 [4]. Thus in a coincidence experiment one would expect to detect a shift of the multiplicity towards higher values when crossing the $p\bar{p}$ pair creation threshold from below. This effect is small since the ratio

$$\frac{\sigma_{inc}^{ch.pion}}{\sigma_{bg}^{ch.pion}}$$

is small and one again would need event numbers of the above order to isolate the signal from statistical fluctuations.

To summarize, the observation of the emission of charged pions (cross section and multiplicity) seems to be better suited than the observation of proton emission to detect the predicted size of the γ -induced $p\bar{p}$ pair creation process. Facilities that could provide sufficient photon flux and are equipped with 4π detectors for pion detection would be CEBAF with the GLAS detector, GRAAL with the BGO detector and ELSA with the SAPHIR detector [14]. An experimental observation of the $p\bar{p}$ pair creation with a reduction of the threshold from $1880 \text{ MeV} \approx 2M$ to values around 1000 MeV would give strong support to the mean field picture of QHD.

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A Calculation of the S-matrix element

In the following we indicate details for the derivation of the S-matrix element of γ -induced $p\bar{p}$ pair creation. The p/\bar{p} wave functions appearing in Eq.(14) are given by

$$\begin{aligned} {}^{(+)}\psi_p^\mu(x) &= \sqrt{\frac{M}{E(\vec{p})V}} u(\vec{p}, \mu) e^{-ipx}, \quad E(\vec{p}) := p_0 = \sqrt{\vec{p}^2 + M^2} \\ {}^{(-)}\psi_{p'}^\rho(x) &= \exp(-iE'x^0) \begin{pmatrix} g(r, E') \omega_\kappa^\rho(\theta, \phi) \\ if(r, E') \omega_{-\kappa}^\rho(\theta, \phi) \end{pmatrix}, \quad E' < 0, \end{aligned} \quad (16)$$

where

$$\omega_\kappa^\rho(\theta, \phi) = \begin{cases} \begin{pmatrix} \frac{\sqrt{j+1-\rho}}{2(j+1)} Y_{j+\frac{1}{2}}^{\rho-\frac{1}{2}}(\theta, \phi) \\ -\frac{\sqrt{j+1+\rho}}{2(j+1)} Y_{j+\frac{1}{2}}^{\rho+\frac{1}{2}}(\theta, \phi) \end{pmatrix}, & \kappa > 0 \\ \begin{pmatrix} \frac{\sqrt{j+\rho}}{2j} Y_{j-\frac{1}{2}}^{\rho-\frac{1}{2}}(\theta, \phi) \\ \frac{\sqrt{j-\rho}}{2j} Y_{j-\frac{1}{2}}^{\rho+\frac{1}{2}}(\theta, \phi) \end{pmatrix}, & \kappa < 0, \end{cases} \quad (17)$$

$$g(r, E') = \begin{cases} a_1 j_{l_\kappa}(Cr), & 0 \leq r \leq r_0, \\ b_1 \sqrt{\frac{2Cr}{\pi}} K_{l_\kappa+\frac{1}{2}}(Cr), & r_0 < r, \end{cases} \quad (18)$$

$$f(r, E') = \begin{cases} a_1 \frac{\kappa}{|\kappa|} \frac{c}{\lambda_1} j_{l_{-\kappa}}(Cr), & 0 \leq r \leq r_0, \\ -b_1 \frac{c}{\lambda_2} \sqrt{\frac{2Cr}{\pi}} K_{l_{-\kappa}+\frac{1}{2}}(Cr), & r_0 < r, \end{cases} \quad (19)$$

$$\begin{aligned} \lambda_1 &:= E' - V + (M - S), & \lambda_2 &:= E' + M, \\ c &:= \sqrt{(E' - V)^2 - (M - S)^2}, & C &:= \sqrt{M^2 - E'^2}, \\ l_\delta &:= \begin{cases} \delta, & \delta > 0, \\ -\delta - 1, & \delta < 0. \end{cases} \end{aligned} \quad (20)$$

The Dirac quantum number κ takes the values

$$\kappa = \pm 1, \pm 2, \pm 3, \dots,$$

whereas j describes the total angular momentum of the \bar{p} state; the functions j_n and $K_{n+\frac{1}{2}}$ are the modified spherical Bessel functions. The normalization constants a_1, b_1 are (up to a phase) determined from

$$\int_V d^3x \left[{}^{(-)}\psi_{p'}^\rho(\vec{x}) \right]^\dagger {}^{(-)}\psi_{p'}^\rho(\vec{x}) = 1. \quad (21)$$

For the evaluation of Eq.(14) we write

$$\begin{aligned} {}^{cor}S_{p,p'}^{1,2;\mu;\rho} &= \frac{ie}{V} [2\pi \delta(E + \bar{E} - k)] \sqrt{\frac{M}{2kE}} \bar{u}(\vec{p}, \mu) \varepsilon_\nu^{1,2} \left[\gamma^\nu + i \frac{\mu}{2M} k'_\lambda \sigma^{\nu\lambda} \right] \times \\ &\quad 4\pi \sum_{l,m} i^l Y_l^{*m}(\hat{k}) \int d^3x j_l(\tilde{k}r) Y_l^m(\theta, \phi) \begin{pmatrix} g(r, E') \omega_\kappa^\rho(\theta, \phi) \\ if(r, E') \omega_{-\kappa}^\rho(\theta, \phi) \end{pmatrix} \end{aligned} \quad (22)$$

with

$$\bar{E} := |E'|, \quad \vec{k} := \vec{k}' - \vec{p},$$

where time integration has been carried out and the exponential $e^{i\vec{k}\cdot\vec{x}}$ is expanded in partial waves. With

$$Y_l^{*m}(\theta, \phi) = (-1)^m Y_l^{-m}(\theta, \phi)$$

and

$$\int d\Omega Y_l^{*m}(\theta, \phi) Y_{l'}^{m'}(\theta, \phi) = \delta_{ll'} \delta_{mm'}$$

the sum in Eq.(22) degenerates to a single contribution and the integral spinor $I(k, \bar{E}, E, \rho)$ defined by

$$I(k, \bar{E}, E, \rho) := 4\pi \sum_{l,m} i^l Y_l^{*m}(\hat{k}) \int d^3x j_l(\tilde{k}r) Y_l^m(\theta, \phi) \begin{pmatrix} g(r, E') \omega_\kappa^\rho(\theta, \phi) \\ if(r, E') \omega_{-\kappa}^\rho(\theta, \phi) \end{pmatrix} \quad (23)$$

is given for $\kappa > 0$ as

$$\begin{aligned} I(k, \bar{E}, E, \rho) = & 4\pi \{ a_1 \int_0^{r_0} dr r^2 \times \\ & \begin{pmatrix} i^{j+\frac{1}{2}} (-1)^{-(\rho-\frac{1}{2})} Y_{j+\frac{1}{2}}^{*-(\rho-\frac{1}{2})}(\hat{k}) \frac{\sqrt{j+1-\rho}}{2(j+1)} j_{j+\frac{1}{2}}(\tilde{k}r) j_{l_\kappa}(cr) \\ i^{j+\frac{1}{2}} (-1)^{-(\rho+\frac{1}{2})+1} Y_{j+\frac{1}{2}}^{*-(\rho+\frac{1}{2})}(\hat{k}) \frac{\sqrt{j+1+\rho}}{2(j+1)} j_{j+\frac{1}{2}}(\tilde{k}r) j_{l_\kappa}(cr) \\ i^{j-\frac{1}{2}+1} \frac{\kappa}{|\kappa|} \frac{c}{\lambda_1} (-1)^{-(\rho-\frac{1}{2})} Y_{j-\frac{1}{2}}^{*-(\rho-\frac{1}{2})}(\hat{k}) \frac{\sqrt{j+\rho}}{2j} j_{j-\frac{1}{2}}(\tilde{k}r) j_{l_{-\kappa}}(cr) \\ i^{j-\frac{1}{2}+1} \frac{\kappa}{|\kappa|} \frac{c}{\lambda_1} (-1)^{-(\rho+\frac{1}{2})} Y_{j-\frac{1}{2}}^{*-(\rho+\frac{1}{2})}(\hat{k}) \frac{\sqrt{j-\rho}}{2j} j_{j-\frac{1}{2}}(\tilde{k}r) j_{l_{-\kappa}}(cr) \end{pmatrix} + \\ & b_1 \int_{r_0}^\infty dr r^2 \sqrt{\frac{2Cr}{\pi}} \times \\ & \begin{pmatrix} i^{j+\frac{1}{2}} (-1)^{-(\rho-\frac{1}{2})} Y_{j+\frac{1}{2}}^{*-(\rho-\frac{1}{2})}(\hat{k}) \frac{\sqrt{j+1-\rho}}{2(j+1)} j_{j+\frac{1}{2}}(\tilde{k}r) K_{l_\kappa+\frac{1}{2}}(Cr) \\ i^{j+\frac{1}{2}} (-1)^{-(\rho+\frac{1}{2})+1} Y_{j+\frac{1}{2}}^{*-(\rho+\frac{1}{2})}(\hat{k}) \frac{\sqrt{j+1+\rho}}{2(j+1)} j_{j+\frac{1}{2}}(\tilde{k}r) K_{l_\kappa+\frac{1}{2}}(Cr) \\ i^{j-\frac{1}{2}+1} \frac{C}{\lambda_2} (-1)^{-(\rho-\frac{1}{2})+1} Y_{j-\frac{1}{2}}^{*-(\rho-\frac{1}{2})}(\hat{k}) \frac{\sqrt{j+\rho}}{2j} j_{j-\frac{1}{2}}(\tilde{k}r) K_{l_{-\kappa}+\frac{1}{2}}(Cr) \\ i^{j-\frac{1}{2}+1} \frac{C}{\lambda_2} (-1)^{-(\rho+\frac{1}{2})+1} Y_{j-\frac{1}{2}}^{*-(\rho+\frac{1}{2})}(\hat{k}) \frac{\sqrt{j-\rho}}{2j} j_{j-\frac{1}{2}}(\tilde{k}r) K_{l_{-\kappa}+\frac{1}{2}}(Cr) \end{pmatrix} \} . \end{aligned} \quad (24)$$

A similar expression results for the case $\kappa < 0$. The integrals in Eq.(24) are evaluated numerically.

Finally, the bilinear expression

$$B(k, \bar{E}, E, \mu, \rho) := \bar{u}(\vec{p}, \mu) \varepsilon_\nu^{1,2} \left[\gamma^\nu + i \frac{\mu_{a.m.}}{2M} k_\lambda \sigma^{\nu\lambda} \right] I(k, \bar{E}, E, \rho) \quad (25)$$

is easily calculated.

B Rosenbluth's formula

Here we show that the influence of static external potentials (V_μ, ϕ) does not alter the form of the free corrected transition current

$$j_{tr,cor}^\mu = \bar{\psi}_{p'} \gamma^\mu \psi_p + \frac{\mu_{a.m.}}{2M} \partial_\nu \bar{\psi}_{p'} \sigma^{\mu\nu} \psi_p , \quad (26)$$

where $\psi_{p'}$ and ψ_p are solutions of the free Dirac equation.

If $\psi_{p'}$ and ψ_p are solutions of the Dirac equation with static scalar and vector potentials, the corrected transition current is required to fulfill the following demands:

- 1) the corrected current has to transform like a Lorentz *vector* under the whole Lorentz group and has to be a bilinear form in the fields $\psi_{p'}$ and ψ_p ,
- 2) it has to be conserved
- 3) and asymptotically, that is $V_\mu, \phi \longrightarrow 0$, given by Eq.(26).

In addition to the bilinear covariants of the Dirac theory we have the four gradient, the potentials themselves and the mass to construct a corrected current. The most general ansatz consistent with **1)** is then

$$j_{tr,cor}^\mu = {}^S F^\mu \bar{\psi}_{p'} \psi_p + {}^V F \bar{\psi}_{p'} \gamma^\mu \psi_p + {}^T F_\nu \bar{\psi}_{p'} \sigma^{\mu\nu} \psi_p \quad (27)$$

where the expressions ${}^S F^\mu$, ${}^V F$, ${}^T F_\nu$ depend on the four Lorentz covariants mentioned only. In accordance with demand **1)** one can split up for instance ${}^S F^\mu$ in the following way

$$\begin{aligned} {}^S F^\mu &= {}^S F^\mu(U^\kappa, \partial^\lambda, \sigma, M) \\ &= \left\{ {}^S F^D \frac{1}{M} \partial^\mu + {}^S F^{D^2 D} \frac{1}{M^3} \partial_\kappa \partial^\kappa \partial^\mu + \mathcal{O}\left(\left(\frac{1}{M}\right)^5\right) \right\} + \\ &\quad \left\{ \frac{1}{M^2} [{}^S F^{\sigma D} \sigma \partial^\mu + {}^S F^{D\sigma} (\partial^\mu \sigma)] + \mathcal{O}\left(\left(\frac{1}{M}\right)^3\right) \right\} + \\ &\quad \left\{ \frac{1}{M} {}^S F^V V^\mu + \frac{1}{M^2} {}^S F^{\sigma V} \sigma V^\mu + \frac{1}{M^3} [{}^S F^{V^2 V} V_\kappa V^\kappa V^\mu + \right. \\ &\quad \left. {}^S F^{\sigma^2 V} \sigma^2 V^\mu] + \mathcal{O}\left(\left(\frac{1}{M}\right)^4\right) \right\} \end{aligned} \quad (28)$$

with similar expressions for ${}^V F$ and ${}^T F_\nu$. The $F^{DD^i}, F^{\dots P^j D^i \dots}, F^{P^i}$ ($P = \sigma, V$) denote dimensionless scalar factors. In Eq.(28) superscripts $DD^i, \dots P^j D^i \dots, P^i$ indicate that in the corresponding curly brackets an expansion with respect to derivatives, derivatives and potentials, potentials only is performed. In the decomposition of ${}^V F$ there also appears a "constant" term ${}^V F^0$.

For the case of ${}^S F^\mu$ and ${}^V F$ the D -terms violate demand **3)** and therefore cannot contribute as well as all the D -terms in the decomposition of ${}^T F_\nu$ except for ${}^T F^{D^1} \partial_\nu$.

If we take the divergence of Eq.(27) the terms with ${}^V F^0$ and ${}^T F^{D^1} \partial_\nu$ vanish by virtue of the equation of motion and the antisymmetry of $\sigma^{\mu\nu}$. The imposition of conditions **2)** and **3)** leads to

$$\begin{aligned} 0 = \partial_\mu j_{tr,cor}^\mu &= \bar{\psi}_{p'} \left(\partial_\mu \overset{\leftrightarrow}{S} \tilde{F}^\mu + \partial_\mu \overset{\leftrightarrow}{V} \tilde{F} \gamma^\mu + \partial_\mu \overset{\leftrightarrow}{T} \tilde{F}_\nu \sigma^{\mu\nu} \right) \psi_p + \\ &\quad \partial_\mu \bar{\psi}_{p'} \left(\overset{\leftrightarrow}{S} \tilde{F}^\mu + \overset{\leftrightarrow}{V} \tilde{F} \gamma^\mu + \overset{\leftrightarrow}{T} \tilde{F}_\nu \sigma^{\mu\nu} \right) \psi_p + \\ &\quad \bar{\psi}_{p'} \left(\overset{\leftrightarrow}{S} \tilde{F}^\mu + \overset{\leftrightarrow}{V} \tilde{F} \gamma^\mu + \overset{\leftrightarrow}{T} \tilde{F}_\nu \sigma^{\mu\nu} \right) \partial_\mu \psi_p, \end{aligned} \quad (29)$$

where the symbol " \leftrightarrow " stands for the right arrangement of the unsaturated derivatives with respect to the product rule, for example

$$\partial_\kappa \partial^\kappa := \overset{\leftarrow}{\partial}_\kappa \overset{\leftarrow}{\partial}^\kappa + 2 \overset{\leftarrow}{\partial}_\kappa \overset{\rightarrow}{\partial}^\kappa + \overset{\rightarrow}{\partial}_\kappa \overset{\rightarrow}{\partial}^\kappa, \quad (30)$$

and " ~ " indicates that we already have omitted the terms forbidden by **3**) and those with ${}^V F^0$ and ${}^T F^{D^1} \partial_\nu$ ($\nu = 0, \dots, 3$).

Each term in the sums over μ in Eq.(29) must vanish by itself; since $\bar{\psi}_{p'}$ and ψ_p are arbitrary states of the spectrum, the corresponding sum of the bilinears in the brackets must vanish. But because of the linear independence of the Dirac matrices $\mathbf{1}, \gamma^\mu, \sigma^{\mu\nu}$

$$\partial_\mu^S \tilde{F}^\mu = \partial_\mu^V \tilde{F} \gamma^\mu = \partial_\mu^T \tilde{F}_\nu \sigma^{\mu\nu} = {}^S \tilde{F}^\mu = {}^V \tilde{F} = {}^T \tilde{F}_\nu = 0 \quad \forall \mu, \nu \quad (\text{no sum over } \mu \text{ and } \nu) . \quad (31)$$

To summarize we have found that in the presence of static external scalar and vector potentials Rosenbluth's formula for the transition current still holds.

C Cross sections

With the calculated S-matrix element of Eq.(22) the corresponding differential cross section $\frac{d\sigma}{d\Omega}$ can be easily obtained. Here we average over the two photon polarizations 1, 2 and sum over the angular momentum projections of the p and \bar{p} states. With phase space $V \frac{d^3 p}{(2\pi)^3}$ and 1 for the continuum p and bound \bar{p} state, respectively, the differential cross section is given as

$$\begin{aligned} \frac{d\sigma}{d\Omega} &= \sum_{1,2;\mu;\rho} \frac{1}{2} \frac{e^2}{V^2} 2\pi \delta(E + \bar{E} - k) \frac{M}{2kE} |B(k, \bar{E}, E, \mu, \rho)|^2 V^2 \frac{p^2 dp}{(2\pi)^3} \\ &= \sum_{1,2;\mu;\rho} \frac{1}{4} \frac{4\pi\alpha}{kE} 2\pi \delta(E + \bar{E} - k) M |B(k, \bar{E}, E, \mu, \rho)|^2 \frac{p^2 dp}{(2\pi)^3} \\ &= \sum_{1,2;\mu;\rho} \frac{1}{2} \frac{\alpha}{kE} \delta(E + \bar{E} - k) M |B(k, \bar{E}, E, \mu, \rho)|^2 \frac{p E dE}{(2\pi)} \\ &= \sum_{1,2;\mu;\rho} \frac{1}{4} \frac{\alpha}{\pi k} M |B(k, \bar{E}, E, \mu, \rho)|^2 p|_{E=k-\bar{E}} \\ &= \sum_{1,2;\mu;\rho} \frac{1}{4} \frac{\alpha}{\pi k} M |B(k, \bar{E}, \mu, \rho)|^2 \sqrt{(k - \bar{E})^2 - M^2} , \end{aligned} \quad (32)$$

where

$$\alpha \approx \frac{1}{137} .$$

The total cross section σ is obtained by (numerically) integrating Eq.(32) over the angle θ and multiplying with 2π .

Inclusive cross sections are obtained by summing over all possible final state contributions at a given γ -energy k with

$$\sigma_{inc}(k) = \sum_{\substack{\bar{E} \\ \bar{E}+E=k \\ E \geq M}} \sigma(\bar{E}, k) . \quad (33)$$

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Figure 1: Schematic view of photon induced $p\bar{p}$ pair creation in the presence of a scalar-vector mean field potential.

Figure 2: Negative energy spectrum of the Dirac equation Eq.(5) with the mean field potential parameters $S = 400$ MeV, $V = 450$ MeV and the nuclear radius $r_0 = 7.2$ fm corresponding to ^{208}Pb .

Figure 3: Dependence of the cross section σ for direct p emission due to γ -induced $p\bar{p}$ pair creation on the potential depth $S = V + 50$ MeV in steps of 10 MeV. The energy of the proton is fixed at $M + 8$ MeV (M is the nucleon mass) while the negative energy solution is in the highest $\kappa = +1$ state corresponding to a \bar{p} in the lowest lying $\kappa = -1$ state. The radius $r_0 = 7.2$ fm adjusted to ^{208}Pb is kept fixed.

Figure 4: Differential inclusive cross section $\frac{d\sigma_{inc}}{d\Omega}$ for direct p emission due to $p\bar{p}$ pair creation at a photon energy of $k = k_{th} + 50$ MeV (dashed) and of $k = k_{th} + 30$ MeV (solid) with the threshold energy $k_{th} = 1035$ MeV. Nuclear potential parameters are $S = 450$ MeV, $V = 400$ MeV and $r_0 = 7.2$ fm for ^{208}Pb .

Figure 5: Inclusive $p\bar{p}$ pair creation cross section σ_{inc} including negative energy states up to $\kappa = +7$ in dependence on the photon energy k . Mean field potential parameter values are as in Fig. 4. The threshold for photoproduction of $p\bar{p}$ pairs lies for these potential parameters at $k_{th} = 1035$ MeV.

Figure 1:

Figure 2:

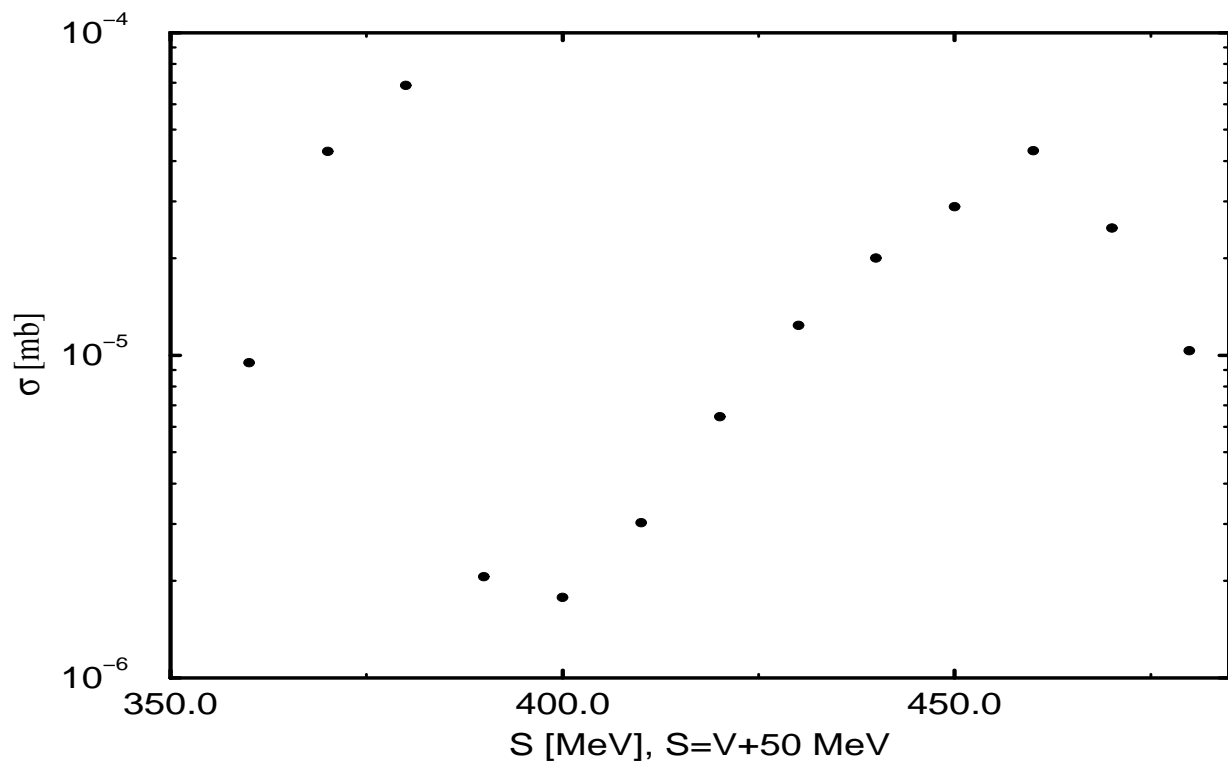


Figure 3:

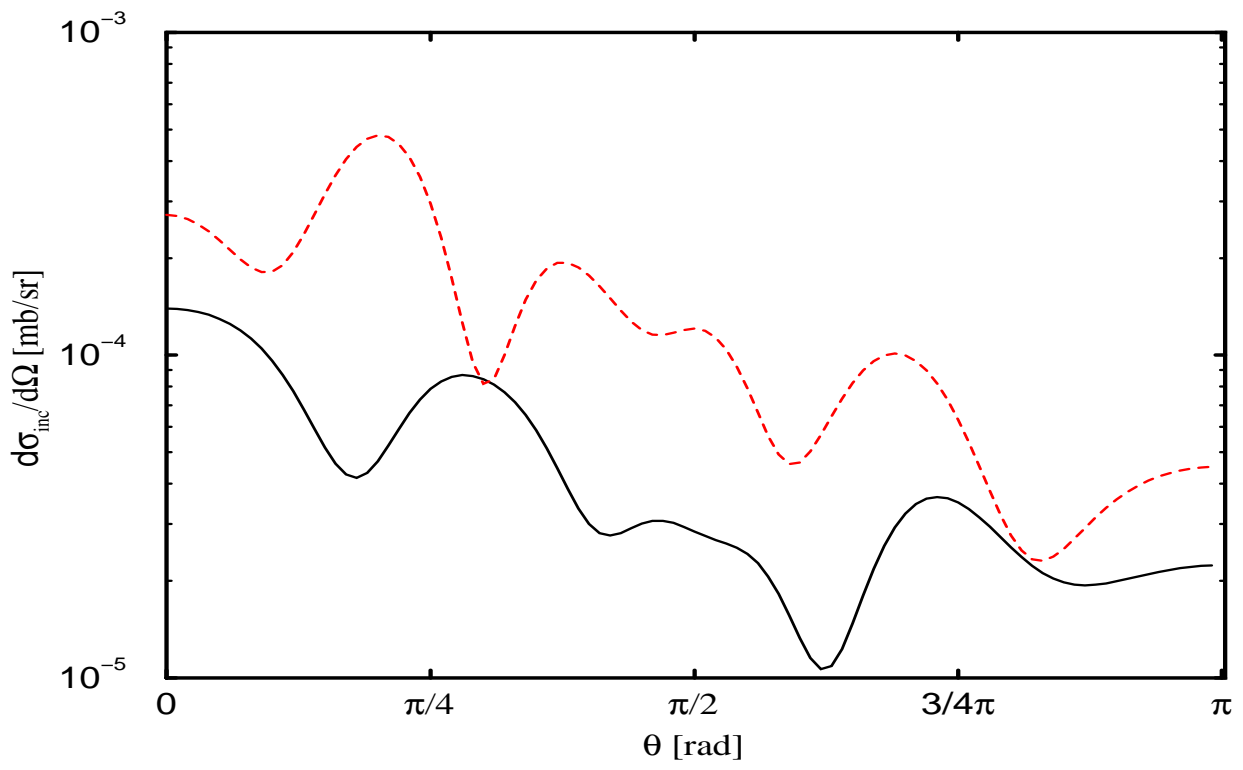


Figure 4:

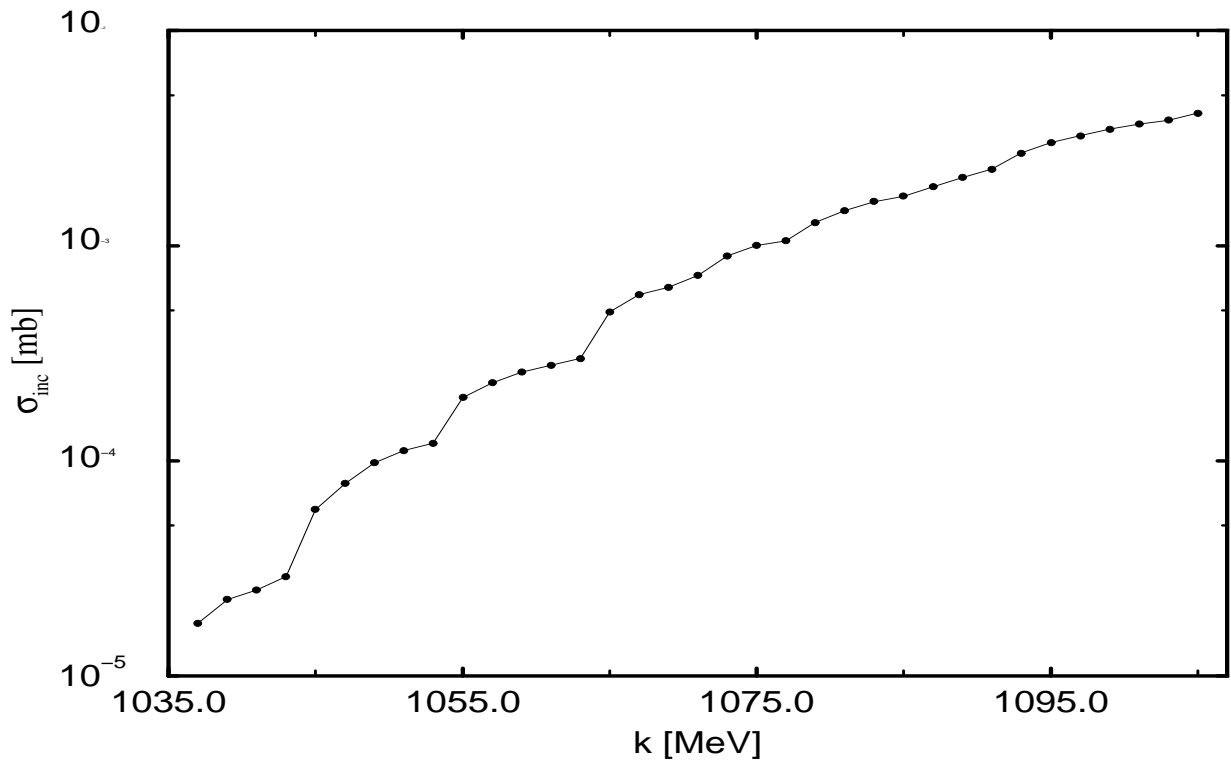


Figure 5:

